RATE AND DETECTION ERROR-EXPONENT TRADEOFFS OF JOINT COMMUNICATION AND SENSING

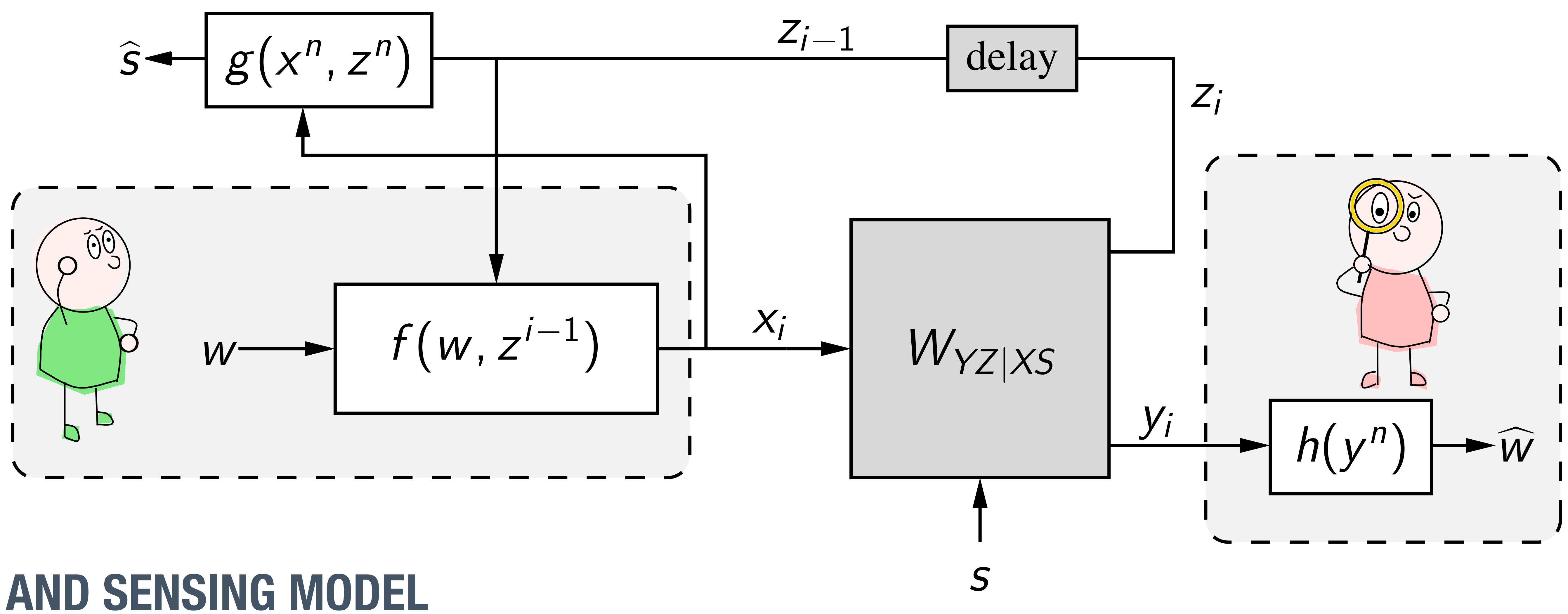
2nd IEEE International Hybrid Symposium on Joint Communications & Sensing

Meng-Che Chang, Tuna Erdogan, Shi-Yuan Wang, Matthieu R. Bloch
School of Electrical and Computer Engineering
Georgia Institute of Technology

- **GENERAL CONTEXT**
  - mmWave systems enable convergence of radar and communication wavelengths
  - Previously separated communication and sensing systems can coexist on a single hardware
  - Sensing can significantly improve communication performance

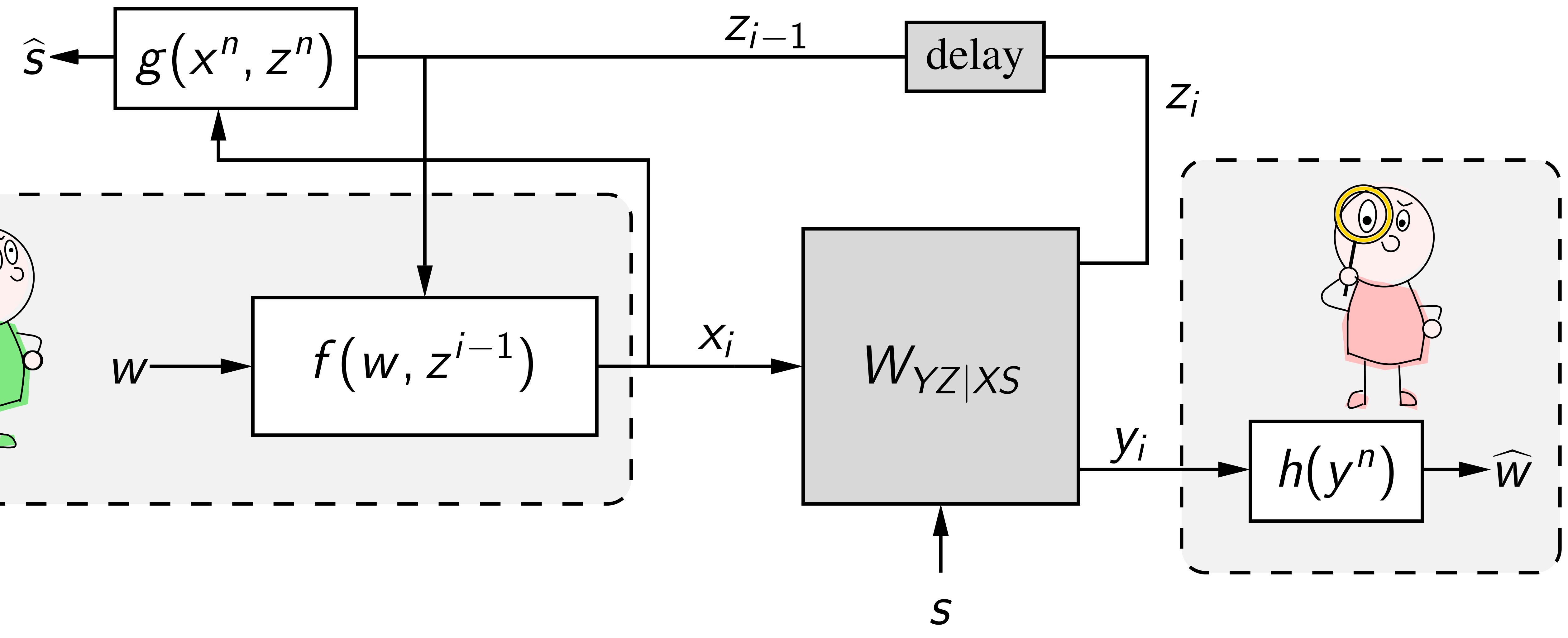
- **INFORMATION-THEORETIC PERSPECTIVE ON JOINT COMMUNICATION AND SENSING**
  - **Objective:** develop fundamental insights into benefits of joint approach
  - **Previous works:** sensing independent and identically distributed (i.i.d.) channel state
    - **[Zhang et al.'11,Kobayashi et al.'18'19, Ahmadipour et al.'21]**
    - Characterization of rate-distortion region with (optimal) open-loop strategies

- **PRESENT WORK: MODEL WITH MEMORY**
  - Discrete Memoryless Channel (DMC) with **fixed** channel state
    - **Motivation:** Channel change rate much slower than communication rate
  - Full characterization of sensing vs. communication tradeoff in open loop
  - Identification of significant benefits when **adapting** to channel with sensing



- **JOINT COMMUNICATION AND SENSING MODEL**
  - State-dependent Discrete Memoryless Channel (Compound Channel)
  - State $s \in \mathcal{S}$, $|\mathcal{S}| < \infty$
  - Encoder: $f_i : [1; M] \times \mathcal{Z}^{i-1} \to \mathcal{X} \ \forall i \in [1; n]$, decoder: $h : \mathcal{Y}^n \to [1; M]$, estimator: $g : \mathcal{X}^n \times \mathcal{Z}^n \to \mathcal{S}$

- **PERFORMANCE METRICS**
  - Communication-error probability: $P_{\text{c}}^{(n)} \triangleq \max_{s \in \mathcal{S}} \max_{w \in [1;M]} \mathbb{P}(h(Y^n) \neq w | W = w, S = s)$
  - Detection-error probability: $P_{\text{d}}^{(n)} \triangleq \max_{s \in \mathcal{S}} \frac{1}{M} \sum_{w=1}^{M} \mathbb{P}(g(Z^n) \neq s | S = s, W = w)$
  - Rate: $R \triangleq \frac{1}{n} \log M$ and detection error exponent $E_{\text{d}}^{(n)} \triangleq -\frac{1}{n} \log P_{\text{d}}^{(n)}$



- **STRATEGIES: OPEN LOOP VS CLOSED LOOP**
  - Open loop model does not use channel state information
  - Closed loop model uses feedback for encoding
    - Adapt communication to sensing

- **EXISTENCE OF TRADEOFFS BETWEEN PERFORMANCE METRICS**
  - No tradeoff between rate and estimation error: possible to transmit at capacity with vanishing estimation error
  - Tradeoff is between rate and **detection error exponent**
  - **Intuition:** both rate and detection error exponent depend on **type** of codewords

THEOREM: RATE AND EXPONENT REGION FOR OPEN-LOOP

Joint communication and sensing region

$$\mathcal{C}_{\text{open}} = \bigcup_{P_X \in \mathcal{P}_X} \left\{ \begin{array}{l} (R, E) \in \mathbb{R}_+^2 : \\ R \leqslant \min_{s \in \mathcal{S}} \mathbb{I}(P_X, W_{Y|X,s}) \\ E \leqslant \phi(P_X) \end{array} \right\}$$

where

$$\phi(P_X) = \min_{s \in \mathcal{S}} \min_{s' \neq s} \max_{\ell \in [0,1]} - \sum_x P_X(x) \log \left(\sum_z W_{Z|x,s}(z)^\ell W_{Z|x,s'}(z)^{1-\ell} \right)$$

COROLLARY: NO TRADEOFF CONDITION

If there exists $x_0 \in \mathcal{X}$ such that for all $x \in \mathcal{X}$ there exists a permutation π_x on $\mathcal{Z}$ such that for every $s \in \mathcal{S}$

$$W_{Z|X,s}(z|x) = W_{Z|X,s}(\pi_x(z)|x_0)$$

then

$$\mathcal{C}_{\text{open}} = \left\{ \begin{array}{l} (R, E) \in \mathbb{R}_+^2 : \\ R \leqslant \max_{P_X} \min_{s \in \mathcal{S}} \mathbb{I}(P_X, W_{Y|X,s}) \\ E \leqslant \max_{P_X} \phi(P_X) \end{array} \right\}$$

- **STATE-DEPENDENT PERFORMANCE METRICS**
  - State-dependent communication-error probability: $P^{(n)}_{\mathrm{c},s} \triangleq \max_w \mathbb{P}(h(Y^n) \neq w | W = w, S = s)$
  - State-dependent detection-error probability: $P^{(n)}_{\mathrm{c},s} \triangleq \max_w \mathbb{P}(h(Y^n) \neq w | W = w, S = s)$
  - State-dependent detection-error exponent: $E^{(n)}_{\mathrm{d},s} \triangleq -\frac{1}{n} \log P^{(n)}_{\mathrm{d},s}$

THEOREM: INNER BOUND FOR RATE AND EXPONENT REGION FOR CLOSED-LOOP

Joint communication and sensing region

$$\mathcal{C}^s_{\text{closed}} \supseteq \bigcup_{P_X \in \mathcal{P}_X} \left\{ \begin{array}{l} (R, E) \in \mathbb{R}^2_+ : \\ R \leqslant \mathbb{I}(P_X, W_{Y|X,s}) \\ E \leqslant \psi_s(P_X) \end{array} \right\}$$

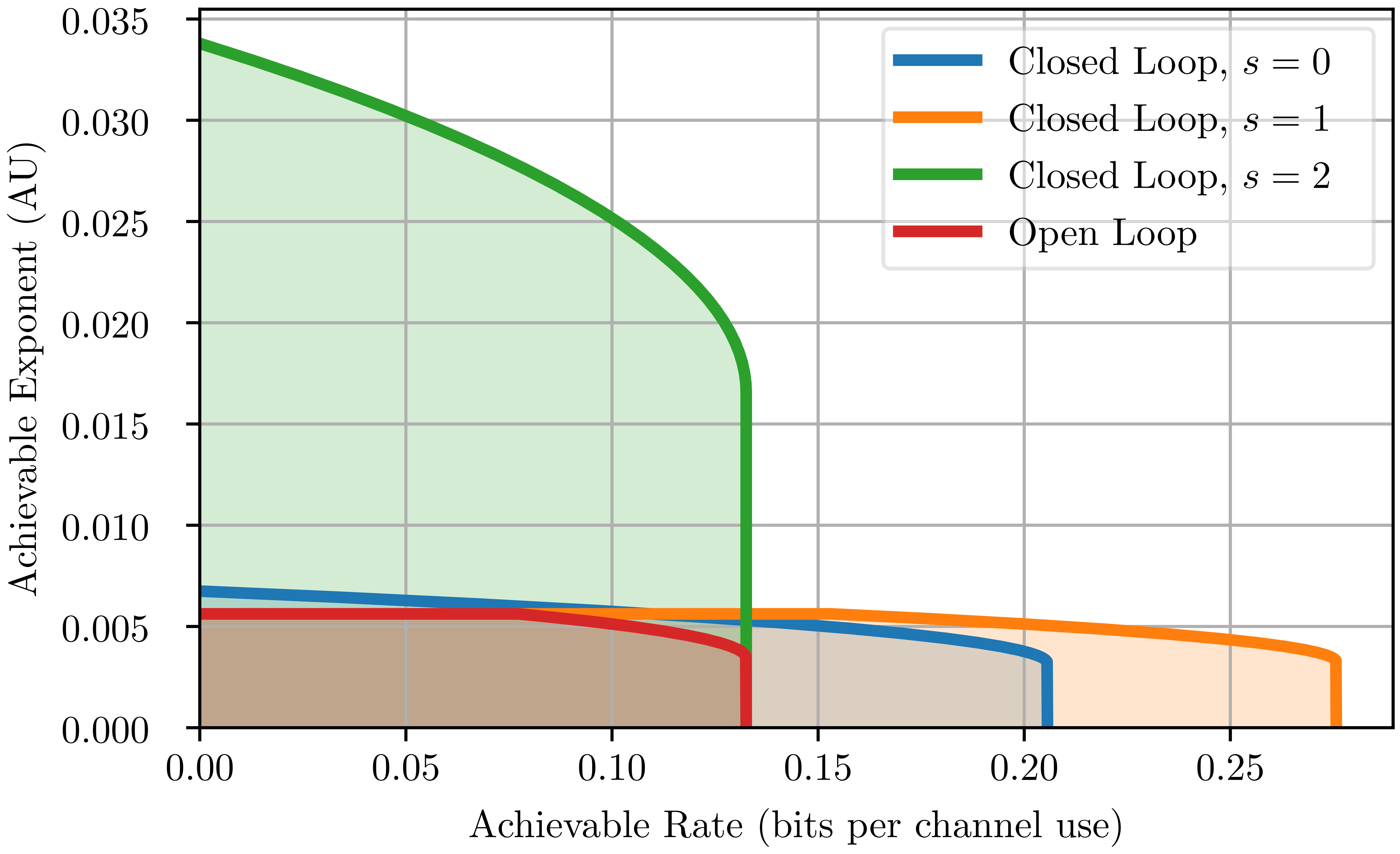
where

$$\psi_s(P_X) = \min_{s' \neq s} \max_{\ell \in [0,1]} -\sum_x P_X(x) \log \left(\sum_z W_{Z|X,s}(z|x)^{\ell} W_{Z|X,s'}(z|x)^{1-\ell} \right)$$

- **SIMULATION RESULTS FOR OPEN LOOP AND CLOSED LOOP REGIONS**

S \ X	0	1
0	0.9	0.3
1	0.8	0.2
2	0.7	0.2

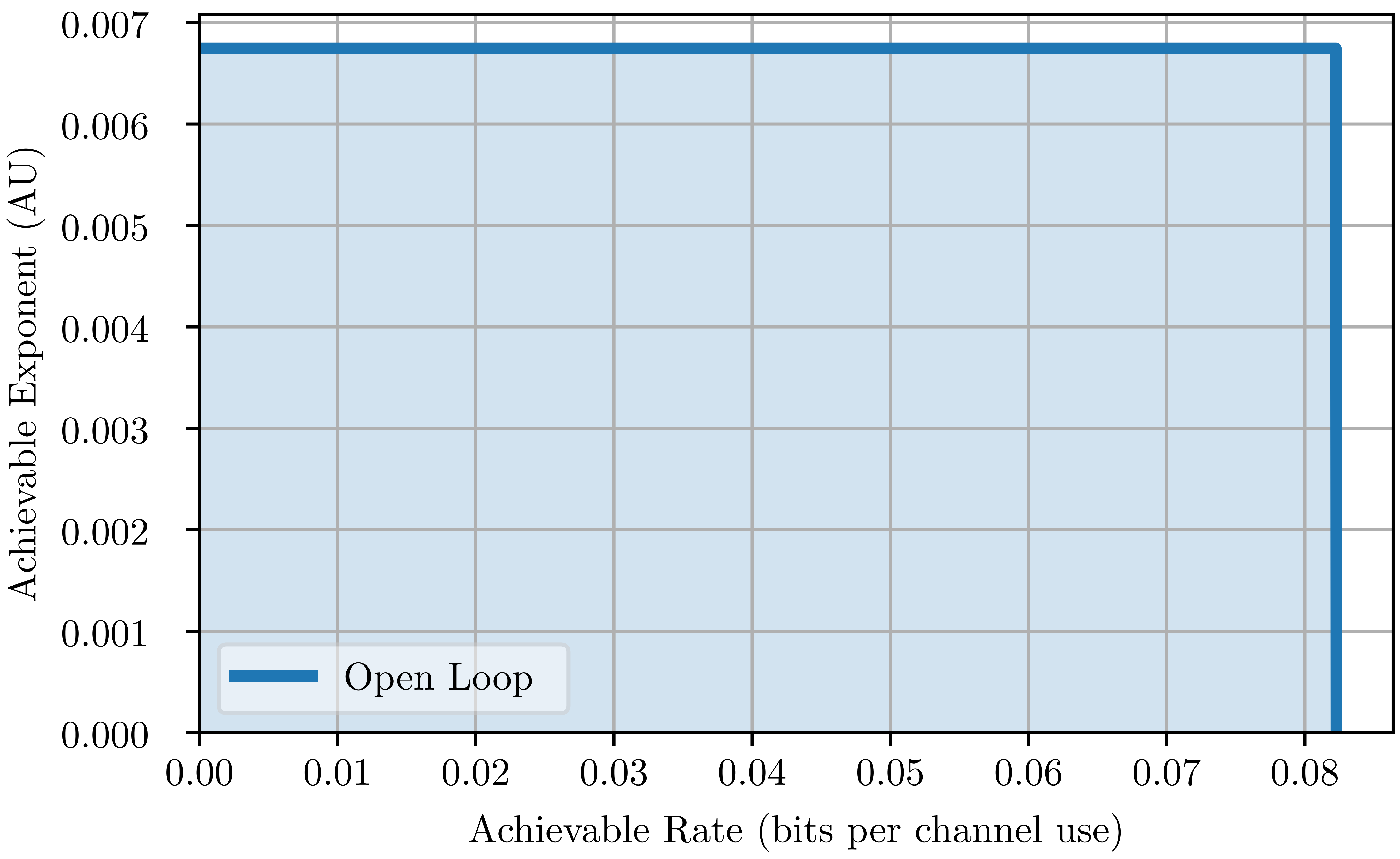
Table 1: Table for $W_{Z|X,S}(0) = W_{Y|X,S}(0)$ for all $X \in \{0,1\}$ and $S \in \{0,1,2\}$.



SIMULATION RESULT FOR NO TRADEOFF EXAMPLE

S \ X	0	1
0	0.9	0.1
1	0.8	0.2
2	0.7	0.3

Table 1: Table for $W_{Z|X,S}(0) = W_{Y|X,S}(0)$ for all $X \in \{0,1\}$ and $S \in \{0,1,2\}$.



DEFINITION: ACHIEVABILITY OF RATE AND EXPONENT PAIR

(R, E) is achievable if for every $\epsilon > 0$, there exists n large enough and a code $\mathcal{C}$ of length n such that

$$P_{\text{c}}^{(n)} \leqslant \epsilon,$$
$$E_{\text{d}}^{(n)} \geqslant E - \epsilon,$$
$$\frac{1}{n}\log|\mathcal{C}| \geqslant R - \epsilon$$

- **SKETCH OF PROOF:**
  - Fix the type of codewords, P_X
  - By the Compound Channel Coding Theorem **[Blackwell et al.'59]**, $R = \min_{s\in\mathcal{S}} \mathbb{I}(P_X, W_{Y|X,s})$ is achievable
  - By Open Loop hypothesis testing **[Naghshvar-Javidi'13, Nitinawarat et al.'13]**, the exponent

$$\phi(P_X) = \min_{s\in\mathcal{S}} \min_{s'\neq s} \max_{\ell\in[0,1]} -\sum_x P_X(x) \log\left(\sum_z W_{Z|x,s}(z)^{\ell} W_{Z|x,s'}(z)^{1-\ell}\right)$$

is asymptotically achievable

SKETCH OF PROOF:

- Assume that (R, E) is achievable
- Since there are at most polynomial number of types, there exists a type set $\mathcal{T}$, such that for all $P_X \in \mathcal{T}$ with subcode $\mathcal{C}_{P_X}$
  - Probability of error for reliable communication is small
  - Subcode includes almost all codes
  - Rate of the subcode is upper bounded by $\min_s \mathbb{I}\left(P_X, W_{Y|X,s}\right)$
- Choose $P_X^* = \operatorname*{argmin}_{P_X \in \mathcal{T}} \phi\left(P_X\right)$
- Then the probability of error is dominated by the probability of error of the subcode
  - Since it has a fixed type the error exponent is bounded by $\phi\left(P_X^*\right)$

- **SKETCH OF PROOF:**
- We first define the three-phase encoder/decoder pairs:
  - During the time $t \in [1; \Delta_1 n]$:
    - $P_X^{\#}$ is a type that maximizes $\phi(P_X)$
    - Select a sequence $\mathbf{v}$ that has the type $P_X^{\#}$
    - Encoder maps any input message $w \in [1; M]$ into $\mathbf{v}$ and the state estimator is a ML detector.
    - This step is to estimate the state by observing the received sequence.
  - During the time $t \in [\Delta_1 n + 1; \Delta_2 n]$
    - Pick a channel code with size $|\mathcal{S}|$.
    - The encoder maps the estimated state from phase 1 into the corresponding codeword.
    - The channel code decoder at the receiver detect the state.
    - This step is to convey the information of the state to the receiver.
  - During the time $t \in [\Delta_2 n + 1; n]$
    - Pick a channel code with size M and has the type P_X ,
      where the rate of the code is related to the estimated state in the phase 1.
    - The encoder maps the message w into the corresponding codeword.
    - The message decoder is the channel code decoder and the state estimator is the ML detector.

- Exact characterization of joint communication and sensing region for open loop as rate and detection-error exponent as off line resource allocation
- Lower bound characterization of the joint communication and sensing region for adaptive
- Clear benefits for using sensing for communication
- **FUTURE EXTENSIONS:**
  - More complicated model
  - Characterization of closed loop performance

Joint Transmission and State Estimation: A Constrained Channel Coding Approach
W. Zhang, S. Vedantam and U. Mitra
IEEE Transactions on Information Theory, 2011

Joint State Sensing and Communication: Optimal Tradeoff for a Memoryless Case
M. Kobayashi, G. Caire and G. Kramer
Proceedings of IEEE International Symposium on Information Theory, Jul. 2019

Joint State Sensing and Communication over Memoryless Multiple Access Channels
M. Kobayashi, H. Hamad, G. Kramer and G. Caire
TBD

An Information-Theoretic Approach to Joint Sensing and Communication
M. Ahmadipour, M. Kobayashi, M. Wigger and G. Caire
TBD

Controlled Sensing for Multihypothesis Testing
S. Nitinawarat, G. K. Atia, and V. V. Veeravalli
IEEE Transactions on Automatic and Control, vol. 58, no.10, Oct. 2013

The Capacity of a Class of Channels
D. Blackwell, L. Breiman and A. J. Thomasian
The Annals of Mathematical Statistics, vol. 30, no. 4, Dec. 1959

Sequentiality and Adaptivity Gains in Active Hypothesis Testing
M. Naghshvar and T. Javidi
IEEE Journal of Selected Topics in Signal Processing, vol. 7, no. 5, Oct. 2013